1.

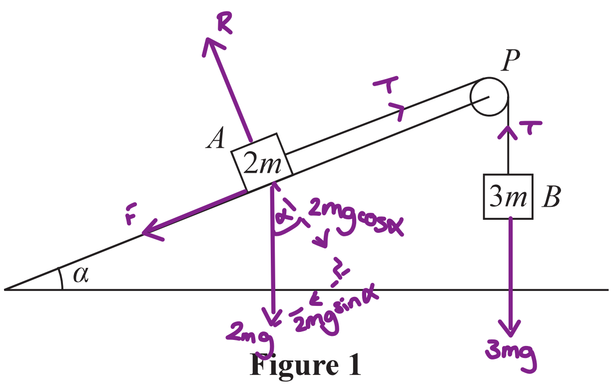


Figure 1

Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle α to the horizontal ground, where $\tan\alpha = \frac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane. Block B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{2}{3}$

$F = \mu R$

$F = \frac{2}{3}R$

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T.

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \dfrac{12mg}{5}$ **(8)**

After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P.

(b) Determine whether A will remain at rest, carefully justifying your answer. **(2)**

(c) Suggest two refinements to the model that would make it more realistic. **(2)**

a) $F = ma$

R(↖):

$R - 2mg\cos\alpha = 0$

$R = 2mg\cos\alpha$ - 1

For A: R(↗) - 1

$T - F - 2mg\sin\alpha = 2ma$

substitute in

$F = \mu R$

$F = \frac{2}{3}R$

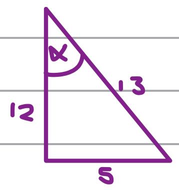
$F = \frac{2}{3}(2mg\cos\alpha)$

a) $T - \frac{2}{3}(2mg\cos\alpha) - 2mg\sin\alpha = 2ma$

$T - \frac{2}{3}\left(\frac{24}{13}mg\right) - \frac{10}{13}mg = 2ma$

$-(2)$

$T - 2mg = 2ma \quad -(1)$



$\tan\alpha = \frac{5}{12}$

$\cos\alpha = \frac{12}{13}$

$\sin\alpha = \frac{5}{13}$

For B R(↓)

$3mg - T = 3ma \quad -(2)$

$a = \frac{3mg - T}{3m} \quad -(2)$

(2) into (1)

$T - 2mg = 2m \times \left(\frac{3mg - T}{3m}\right) \quad -(1)$

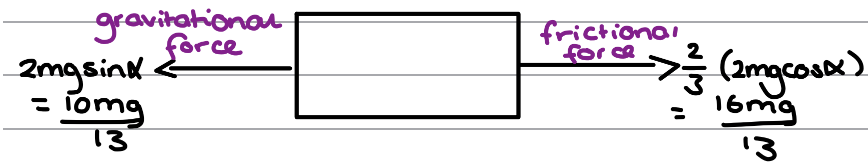
$T - 2mg = \frac{6mg - 2T}{3}$

$3T - 6mg = 6mg - 2T$

$5T = 12mg$

$T = \frac{12mg}{5} \quad -(1)$

b) Forces on A:



$\frac{16mg}{13} > \frac{10mg}{13} \quad -(1)$

∴ frictional force > gravitational force, so A will remain at rest. - (1)

c) • It's unlikely that the pulley is smooth - modelling it as rough would be more realistic - (1)

• It's also not likely the string is light - it must have some mass. - (1)

2.

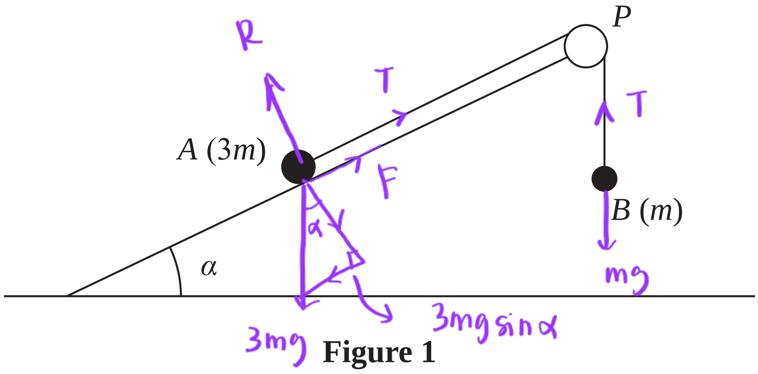


Figure 1

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(2)

(b) show that the acceleration of A is $\frac{1}{10}g$

(7)

(c) sketch a velocity-time graph for the motion of B, from the instant when A is released from rest to the instant just before B reaches the pulley, explaining your answer.

(2)

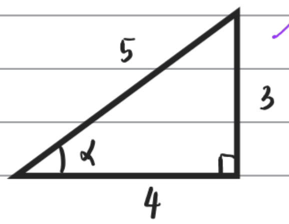
In reality, the string is not light.

(d) State how this would affect the working in part (b).

(1)

Given $\tan\alpha = \frac{3}{4}$

(a)



$\sin\alpha = \frac{3}{5}$ $\cos\alpha = \frac{4}{5}$

Σ forces $= ma$

$3mg\sin\alpha - F - T = (3m)a$

$3mg\sin\alpha - F - T = 3ma$

$\therefore$ Equation of motion for A : $3mg\sin\alpha - F - T = 3ma$ (1)

(1)

(b) $R = 3mg\cos\alpha$ (1)

$= 3mg\left(\frac{4}{5}\right)$

$= \frac{12mg}{5}$ (1)

$F = \mu R$

$= \frac{1}{6} \times \frac{12mg}{5}$ (1)

$= \frac{2mg}{5}$

Motion for B : $T - mg = ma$

$T = mg + ma$ (1)

(1)

Motion for A : $3mg\sin\alpha - F - T = 3ma$

$$3mg\left(\frac{3}{5}\right) - \frac{2mg}{5} - mg - ma = 3ma$$

$$\frac{9mg}{5} - \frac{2mg}{5} - \frac{5mg}{5} = 3ma + ma$$

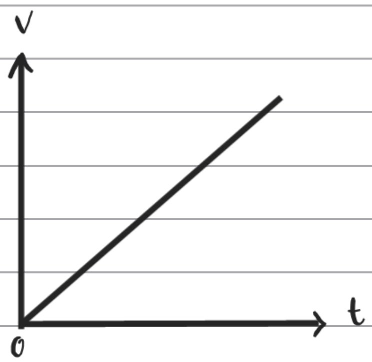
$$\frac{2\cancel{m}g}{5} = 4\cancel{m}a$$

$$\frac{2g}{5} = 4a$$

$$a = \frac{2g}{20} \quad (1)$$

$$a = \frac{1}{10}g \ \# \quad (1)$$

(c)



(1)

Acceleration of B is constant (1)

(d) Tension on A would be different to tension on B (1)